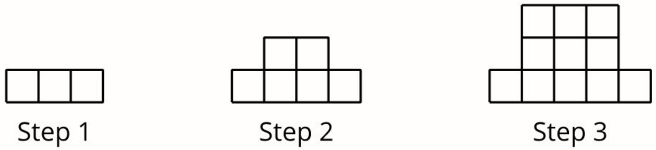
Algebra I Input #5

Quadratic Patterns & Polynomials

Part 1: Exploring the patterns

2.3: Expressing a Growth Pattern



Here is a pattern of squares.

1. Is the number of small squares growing linearly? Explain how you know.

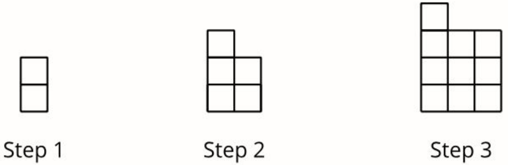
2. Complete the table.

step	number of small squares
1	
2	
3	
4	
5	
10	
12	
n	

3. Is the number of small squares growing exponentially? Explain how you know.

Understanding the pattern…

In this lesson, we saw some quantities that change in a particular way, but the change is neither linear nor exponential. Here is a pattern of shapes, followed by a table showing the relationship between the step number and the number of small squares.



step	total number of small squares
1	2
2	5
3	10
n	$n^2 + 1$

The number of small squares increases by 3, and then by 5, so we know that the growth is not linear. It is also not exponential because it is not changing by the same factor each time. From Step 1 to Step 2, the number of small squares grows by a factor of $\frac{5}{2}$, while from Step 2 to Step 3, it grows by a factor of 2.

Quadratic

From the diagram, we can see that in Step 2, there is a 2-by-2 square plus 1 small square added on top. Likewise, in Step 3, there is a 3-by-3 square with 1 small square added. We can reason that the nth step is an n-by-n arrangement of small squares with an additional small square on top, giving the expression $n^2 + 1$ for the number of small squares.

The relationship between the step number and the number of small squares is a quadratic relationship, because it is given by the expression $n^2 + 1$, which is an example of a **quadratic expression**. We will investigate quadratic expressions in depth in future lessons.

Part 2: Polynomials

A **polynomial** of one variable is a sum of terms in which each term is a constant times a variable raised to a nonnegative integer power. The following are all polynomials:

$$x^5 + 3x^2 + 1, \qquad t^9 - 3t^8 + 276t, \qquad -8z^{10} + z^5 - 1.$$

As the example polynomials above illustrate, we usually write polynomials such that the exponents of the variable decrease from left to right. Polynomials can also have more than one variable. For example, these are also polynomials:

$$2xy, \qquad \frac{x^3}{8} - \frac{y^3}{8}, \qquad 3z^2y - 5zy + 3zy^2 + 2.$$

All variables in polynomials must have nonnegative powers, and the variables can't be in denominators or under square root signs, etc. The following are not polynomials:

$$x + \frac{1}{x}, \qquad x^2 + \frac{x}{y} - 3y^2, \qquad \sqrt{a^2 + b^2}.$$

Nearly all the work we do in this book will be with polynomials that have only one variable. We call the highest power of the variable in such a polynomial the **degree** of the polynomial, and we call the term containing this highest power the **leading term** of the polynomial. For example, the leading term of

$$f(x) = 3x^4 + 2x^2 - 7$$

is $3x^4$. To denote that f has degree 4, we write $\deg f = 4$.

Polynomials

Pick the expression that matches this description:

A 3rd degree binomial with a constant term of 8

Choose 1 answer:

- (A) $-5x^3 + 8$
- (B) $x^3 - x^2 + 8$
- (C) $2x^8 + 3$
- (D) $8x^3 + 2x + 3$

Which polynomials are in standard form?

Choose all answers that apply:

- (A) $10 - n$
- (B) $5n + 3n^3 - 1$
- (C) $n + 4n^2 - 7n^3$
- (D) None of the above