Check Your Understanding

1. (a) $a \div b = a \cdot (\frac{1}{b})$
(b) $c - d = c + (-d)$
(c) $ab = (-a)(-b)$
(d) If $ab = 1$, then $b = \frac{1}{a}$ and $a = \frac{1}{b}$.

2. (a) Because if $a = 0$ is true, then $a = 0$ or $b = 0$ is true, so the proof is complete.
(b) You know this because every real number except 0 has a reciprocal.
(c) The left side becomes b and the right side becomes 0.
(d) This proves the Zero Product Property because it shows that either a or b has to equal zero. That is, if a doesn't equal zero, then b has to equal zero.

On Your Own

3. (a) The definition uses x and y. Replace the x with 4 and y with 6 to get $4 \heartsuit 6 = 3(4) + 6 = 12 + 6 = 18$
(b) $6 \heartsuit 4 = 3 \cdot 6 + 4 = 18 + 4 = 22$
(c) Look at the solutions to parts a and b to see that $4 \heartsuit 6 \neq 6 \heartsuit 4$. So $\heartsuit$ is not commutative.

4. (a) Yes, $\spadesuit$ is commutative. You can check by seeing if $-3(x + y) = -3(y + x)$ is true. And, since the two numbers are inside the parentheses, and they are added (and addition is commutative), then $\spadesuit$ is also commutative.

(b) No, ♠ isn't associative. It's a little tougher to show this using variables, so pick some numbers. You need three, so try 1, 2, and 3.
First, test (1♠2)♠3.

$$
\begin{aligned}
(1♠2) &= -3(1+2) \\
&= -3(3) \\
&= -9
\end{aligned}
$$

So substitute −9 for 1♠2, and continue.

$$
\begin{aligned}
(-9)♠3 &= -3((-9)+3)) \\
&= -3(-6) \\
&= 18
\end{aligned}
$$

So (1♠2)♠3 = 18. Now, test 1♠(2♠3).

$$
\begin{aligned}
(2♠3) &= -3(2+3) \\
&= -3(5) \\
&= -15
\end{aligned}
$$

So substitute −15 for 2♠3, and continue.

$$
\begin{aligned}
1♠(-15) &= -3(1+(-15)) \\
&= -3(-14) \\
&= 42
\end{aligned}
$$

In order for the operation to be associative, it has to be true for any numbers you choose. Since $18 \neq 42$, you have shown that ♠ is not associative.