M8 Inputs 1 & 2

Square Roots & Pythagorean Theorem

Part 1: Square Roots & Exponents

What two whole numbers does each square root lie between? Be prepared to explain your reasoning.

1. $\sqrt{7}$

2. $\sqrt{23}$

Decide if each statement is true or false.

$\left(\sqrt{5}\right)^2 = 5$

$\left(\sqrt{10}\right)^2 = 100$

$\left(\sqrt{9}\right)^2 = 3$

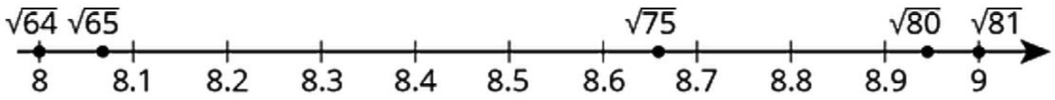
$\left(\sqrt{16}\right) = 2^2$

$7 = \left(\sqrt{7}\right)^2$

Square Roots Summary

In general, we can approximate the values of square roots by observing the whole numbers around it, and remembering the relationship between square roots and squares. Here are some examples:

- $\sqrt{65}$ is a little more than 8, because $\sqrt{65}$ is a little more than $\sqrt{64}$ and $\sqrt{64} = 8$.
- $\sqrt{80}$ is a little less than 9, because $\sqrt{80}$ is a little less than $\sqrt{81}$ and $\sqrt{81} = 9$.
- $\sqrt{75}$ is between 8 and 9 (it's 8 point something), because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately 8.67, because $8.67^2 = 75.1689$.

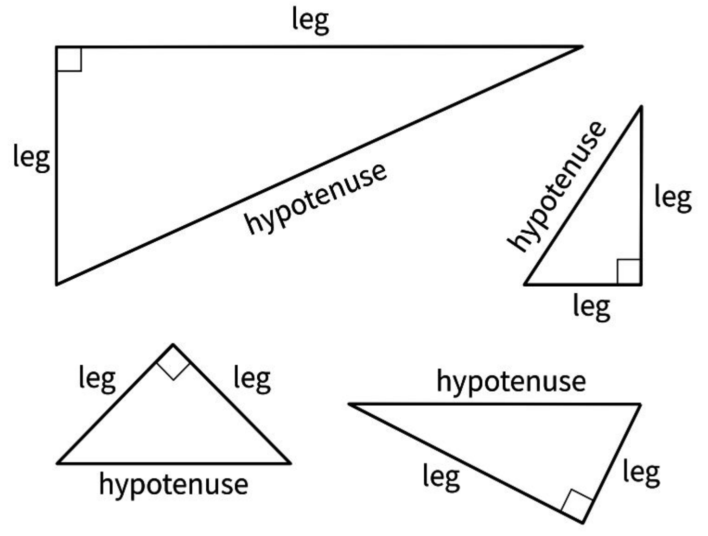


If we want to find a square root between two whole numbers, we can work in the other direction. For example, since $22^2 = 484$ and $23^2 = 529$, then we know that $\sqrt{500}$ (to pick one possibility) is between 22 and 23.

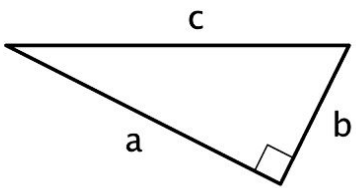
Many calculators have a square root command, which makes it simple to find an approximate value of a square root.

Part 2: Right Triangles & Pythagorean Theorem

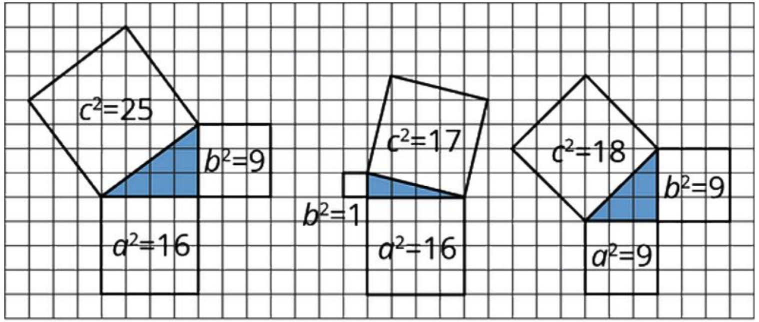
A *right triangle* is a triangle with a right angle. In a right triangle, the side opposite the right angle is called the **hypotenuse**, and the two other sides are called its **legs**. Here are some right triangles with the hypotenuse and legs labeled:



We often use the letters a and b to represent the lengths of the shorter sides of a triangle and c to represent the length of the longest side of a right triangle. If the triangle is a right triangle, then a and b are used to represent the lengths of the legs, and c is used to represent the length of the hypotenuse (since the hypotenuse is always the longest side of a right triangle). For example, in this right triangle, $a = \sqrt{20}$, $b = \sqrt{5}$, and $c = 5$.



Here are some right triangles:



Notice that for these examples of right triangles, the square of the hypotenuse is equal to the sum of the squares of the legs. In the first right triangle in the diagram, $9 + 16 = 25$, in the second, $1 + 16 = 17$, and in the third, $9 + 9 = 18$. Expressed another way, we have

$$a^2 + b^2 = c^2$$

This is a property of all right triangles, not just these examples, and is often known as the **Pythagorean Theorem**. The name comes from a mathematician named Pythagoras who lived in ancient Greece around 2,500 BCE, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures including Babylon, India, and China. In China, a name for the same relationship is the Shang Gao Theorem. In future lessons, you will learn some ways to explain why the Pythagorean Theorem is true for *any* right triangle.

Pythagorean Theorem -- Only Right Triangles!

It is important to note that this relationship does not hold for *all* triangles. Here are some triangles that are not right triangles, and notice that the lengths of their sides do not have the special relationship $a^2 + b^2 = c^2$. That is, $16 + 10$ does not equal 18, and $2 + 10$ does not equal 16.